OPTIMAL MULTIPRODUCT PRICES

IN DIFFERENTIATED PRODUCT MARKETS*

María J. Moral

Universidad de Vigo

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Abstract

This paper is devoted to the specification of price equations based on optimal pricing rules for multiproduct firms selling in vertically differentiated markets. The theoretical framework predicts how equilibrium prices depend on the location of products across segments, the firm market share and the firm specialization.

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[a] Dpto. de Economía Aplicada, Campus Universitario As Lagoas s/n , 32004 Ourense (Spain). Tel.: 34 988 36 87 60, Fax: 34 988 36 89 23. e-mail: mjmoral@uvigo.es

1. Introduction

This paper is devoted to the specification of price equations based on optimal pricing rules for multiproduct firms that sell their products in vertically differentiated segments of a market. Drawing on the work of Berry (1994) and Berry, Levinsohn and Pakes (1995) general pricing rules for multiproduct firms have been obtained. From a rather general model for a product-differentiated industry (a nested multinomial logit), I solve the first order conditions for profit maximization problem obtaining the equilibrium relationships among prices, marginal costs and markups.

The main contribution of the paper is to calculate structural expressions for markups fixed by asymmetric multiproduct firms. The segmented and differentiated product market presents a general situation in the sense that the segmentation is independent of firms, that is, segments are compounded by products belonging to different firms which locate their products in any segment. Anderson, De Palma and Thisse (1992) present a particular case of this since they analyze the optimal pricing decision for symmetrical multiproduct firms selling in a segmented market whose segmentation matches up exactly with firms. On the other hand, two extreme cases that are more commonly found in differentiated-product markets are also included. I refer to these situations as *minimum differentiation* (when a firm locates all its products in a market segment) and *maximum differentiation* (when a firm locates each of its products in a different segment). The theoretical framework predicts that firms will fix prices setting unit markups over the marginal costs and that in general markups present two components: a product-specific component that will be greater the higher the segment in which product is; and a firm-specific component that will be greater the higher the segment in which the firm is specialized, and the larger the firm sales are with respect to total sales in the product segment.

The paper is organized as follows. Section two presents the optimal multiproduct prices rules in a segmented market and the predicted markups. Conclusions are in section three.

2. Optimal multiproduct prices

Let be F multiproduct firms that produce J_f substitutive and exclusive variants and sell these products in vertically differentiated segments of a market. Firms set prices that maximize the present value of the expected stream of profits. Assuming that demands depend on contemporary prices and marginal costs are constant and separable across products, the optimisation problem for firm *f* can be solved separately at each period as:

$$\underset{\{p_1^f,\dots,p_{J_f}^f\}}{Max} \sum_{j=1}^{J_f} \left(p_j^f - c_j^f\right) M s_j^f(P) \qquad \forall f = 1,\dots,F \qquad (1)$$

where p_j^f is the price, c_j^f the marginal cost and $s_j^f(P)$ the market share of good *j* produced by the firm *f*, *M* is the exogenous market size and *P* is a *J* order vector compounded by prices of all market products, $P = (p_1^1,\dots,p_{J1}^1,\dots,\dots,p_1^F,\dots,p_{JF}^F)$.

Assuming Bertrand-Nash behaviour[1] and applying the first order conditions the markup for good *j* produced by firm *f* is (from here on the firm superscript is dropped),

$$p_j - c_j = \frac{1}{\eta_{jj}} + \sum_{\substack{k=1 \\ k \neq j}}^{J_f} (p_k - c_k)\frac{s_k \eta_{kj}}{s_j \eta_{jj}} \qquad \forall j = 1,\dots,J_f \qquad (2)$$

where $\eta_{jj} = -\frac{\partial s_j}{\partial p_j}\frac{1}{s_j}$ and $\eta_{kj} = -\frac{\partial s_k}{\partial p_j}\frac{1}{s_k}$ are the demand semielasticities. As products are substitutive, firms fix prices above the value implied by the inverse of the own-price demand semielasticity because they take into account the cross-price effects among all their products.

The solution of the whole system involved for all firms gives the vector of equilibrium prices,

$$P = C + \Psi^{-1} S \qquad (3)$$

where *P*, *C*, and *S* are *J* order column vectors with prices, marginal costs and shares of all market products (notice that $J = \sum_{f=1}^{F} J_f$). Finally, Ψ is a *J*x*J* matrix diagonal in blocks, each diagonal block *f* containing a J_f order matrix of price effects with elements defined as $\psi_{jk} = -\frac{\partial s_k}{\partial p_j}$ $\forall\ j,k = 1,\dots,J_f$, and the other blocks being zero due to the assumed Bertrand behaviour).

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[1] Caplin and Nalebuff (1991) show the existence of a Nash equilibrium with uniproduct firms and pure strategies. Anderson and De Palma (1992) give conditions under which this result can be extended to multiproduct firms.

From the expression (3) it is clear that by specifying a demand system it will be possible to obtain an explicit expression for markups and therefore for equilibrium prices. The discrete choice demand model considered here draws on the work of Berry (1994) and Berry, Levinsohn and Pakes (1995, 1999).[2] I consider a multinomial nested logit with a flexible consumer's utility function in the sense that the price coefficient varies across products.[3] From the standard market shares' formulae for the multinomial nested logit, the diagonal elements of the matrix of price effects are $\psi_{jj} = \alpha_j s_j \left[\frac{1-\sigma s_{j/g}}{1-\sigma} - s_j \right]$, where α_j is the price coefficient, s_j is the market share of product j, $s_{j/g}$ represents the share of product j within its segment g (*within segment share*), and parameter σ measures the degree of product homogeneity within the segment (degree of correlation).[4] The elements outside the diagonal are different according to whether the concerned cross-price effect corresponds with two products belonging to the same segment, $\psi_{jk} = -\alpha_j s_k \left[\frac{\sigma s_{j/g}}{1-\sigma} + s_j \right]$, or to different segments, $\psi_{jk} = -\alpha_j s_j s_k$. The expression for the markup of product j is calculated by multiplying the inverse of this price effects matrix by the vector of market shares. The most general case, presented in the Appendix, is characterised firstly by firms that locate their products in any segment, and secondly by segments that are made up of products from any of the firms. Here just two extreme cases are presented. Firstly, when the firm locates all its products in a segment this is called *minimum differentiation*.[5] Secondly, when the firm locates each of its products in a different segment it is called *maximum differentiation*.

Proposition 1:

With *minimum differentiation* in the market the substitution among all firm products is equal and the markup of product j produced by firm f and belonging to the segment g is,

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[2] See Nevo (2000) for a further discussion.

[3] Jaumandreu and Moral (2002) find that the income effect can be analyzed using a price coefficient varying by segments. Moreover, there is evidence that the disutility for paying a price changes along the product life cycle (Moral and Jaumandreu, 2001). It suggests that a price coefficient varying across product is a sufficiently flexible specification.

[4] A larger degree of correlation implies a higher intensity in price competition. Heteroskedastic nested logit models assume different σ among segments (see McFadden, 1981).

[5] The particular case in which every market segment strictly coincides with the set of products belonging to one firm is analysed in Anderson, De Palma and Thisse (1992).

$$m_j = (1-\sigma)\left[\frac{1}{\alpha_j} + \left(1 + \left(\frac{\sigma}{1-\sigma}\right)\frac{s_{j/g}}{s_j}\right)\left(\frac{\sum_{r=1}^{J_f}\frac{s_r}{\alpha_r}}{\frac{1-\sigma\sum_{r=1}^{J_f}s_{r/g}}{1-\sigma} - \sum_{r=1}^{J_f}s_r}\right)\right] \quad \forall\, j = 1,\ldots,J_f \quad (4)$$

This equation is obtained from the expression [A.3] in the Appendix and taking into account that all firm products (j=1,…,J_f) belong to the same segment g.

Corollary 1:

When the market segmentation matches up exactly with firms (that is, every segment is compounded exclusively by products belonging to the same firm) the markup expression is,

$$m_j = (1-\sigma)\left[\frac{1}{\alpha_j} + \left(1 + \left(\frac{\sigma}{1-\sigma}\right)\frac{s_{j/g}}{s_j}\right)\left(\frac{\sum_{r=1}^{J_f}\frac{s_r}{\alpha_r}}{1 - \sum_{r=1}^{J_f}s_r}\right)\right] \quad \forall\, j = 1,\ldots,J_f \quad (5)$$

Proposition 2:

With *maximum differentiation* in the market (the firm locates each one of its products in a different segment) the markup of product j produced by firm f and belonging to the segment g is,

$$m_j = \left[\frac{1-\sigma}{1-\sigma s_{j/g}}\right]\left[\frac{1}{\alpha_j} + \frac{\sum_{r=1}^{J_f}\left(\frac{1-\sigma}{1-\sigma s_{r/\tilde{g}}}\right)\frac{s_r}{\alpha_r}}{1 - \sum_{r=1}^{J_f}\left(\frac{1-\sigma}{1-\sigma s_{r/\tilde{g}}}\right)s_r}\right] \quad \forall\, j = 1,\ldots,J_f \quad (6)$$

where $\tilde{g}$ represents the market segment of the product r also produced by the firm f. Equation (6) is obtained from the expression [A.3] in the Appendix and taking into account that now the relevant range of variation for segments is g=1,…,J_f.

Maximum differentiation seems to be a more realistic situation in differentiated product markets. Indeed, it seems especially well suited to the automobile industry because the commonly considered segments (small, compact, intermediate,…) consist of sets of similar car models which are produced by different firms. Other examples are the computer industry (desktops, laptops,…), the cornflakes market (chocolate, fibre,…) or the magazine market (travel, music, hobbies,…). In all these markets, firms seem to consciously locate their menu of products across different segments or classes.

Several general comments arise from both propositions. Firstly, a greater degree of homogeneity within segments implies lower markups explaining that effectively there will be more intense price competition. Secondly, it is possible to distinguish terms that depend exclusively on the product (product-specific component) and terms that depend on the firm (firm-specific component). If the parameter α varies across segments as a consequence of the income effect (the higher the segment, the lower it is) then markups will be greater for higher segments because the product-specific component is decreasing in the price coefficient. On the other hand, the product-specific component is increasing in the within segment share of the product therefore a product with higher sales in its segment will have a higher markup. With respect to the firm-specific component, this will be greater when the firm is specialised in higher segments and the market share of the firm is larger. Finally, this framework predicts a dynamic pattern of prices since the evidence suggests that in higher stages of the product life cycle the parameter α is greater, in consequence, we can expect that the product markup will decrease over time.

3. Conclusions

This paper studies the optimal pricing rules for multiproduct firms selling their products in segmented and differentiated product markets. The model follows discrete choice demand models commonly used in literature. The main result of the paper is to obtain the formulae of markups fixed by asymmetric multiproduct firms in a rather general case. Theoretical results predict different markups for every product of the multiproduct firm. In previous works as, for example, Anderson, De Palma and Thisse (1992), identical markups for all firm products were found. Finally, results suggest that together with a product-specific component it is possible to identify a firm-specific component in markups.

Appendix: Markups' derivation in the general case.

Let H_g be the number of products that the firm locates within the segment g $\forall$ g=1,…,G. Then the matrix of price effects for firm f can be written as

$$\psi=\begin{bmatrix} \alpha_1^1 s_1^1\left(\frac{1-\sigma s_{1/1}}{1-\sigma}-s_1^1\right) & \cdots & -\alpha_1^1 s_{H_1}^1\left(\frac{\sigma s_{1/1}}{1-\sigma}+s_1^1\right) & -\alpha_1^1 s_1^1 s_1^2 & \cdots & -\alpha_1^1 s_1^1 s_{H_1}^2 & \cdots & -\alpha_1^1 s_1^1 s_{H_G}^G \\ \cdots & & & & & & & \\ \cdots & & & & & & & \\ -\alpha_{H_1}^1 s_1^1\left(\frac{\sigma s_{H_1/1}}{1-\sigma}+s_{H_1}^1\right) & \cdots & \alpha_{H_1}^1 s_{H_1}^1\left(\frac{1-\sigma s_{H_1/1}}{1-\sigma}-s_{H_1}^1\right) & -\alpha_{H_1}^1 s_{H_1}^1 s_1^2 & \cdots & -\alpha_{H_1}^1 s_{H_1}^1 s_{H_2}^1 & \cdots & -\alpha_{H_1}^1 s_{H_1}^1 s_{H_G}^G \\ -\alpha_1^2 s_1^2 s_1^1 & \cdots & -\alpha_1^2 s_1^2 s_{H_1}^1 & \alpha_1^2 s_1^2\left(\frac{1-\sigma s_{1/2}}{1-\sigma}-s_1^2\right) & \cdots & \cdots & \cdots & -\alpha_{H_1}^1 s_{H_1}^1 s_{H_G}^G \\ \cdots & & & & & & & \\ \cdots & & & & & & & \\ \cdots & & & & & & & \\ -\alpha_{H_G}^G s_1^G s_1^1 & \cdots & -\alpha_{H_G}^G s_{H_G}^G s_{H_1}^1 & -\alpha_{H_G}^G s_{H_G}^G s_1^2 & \cdots & -\alpha_{H_G}^G s_{H_G}^G s_{H_2}^2 & & \alpha_{H_G}^G s_{H_G}^G\left(\frac{1-\sigma s_{H_G/H_G}}{1-\sigma}-s_{H_G}^G\right) \end{bmatrix}$$

[A.1]

All variables have been previously defined in the text. Notice that the J_f products are ordered by segments ($1,\ldots,H_1;1,\ldots,H_2;\ldots;1,\ldots,H_G$), therefore in order to identify every element a superscript has been included indicating the segment (e.g., s_1^1 is the market share of product 1 belonging to segment 1).

Let us decompose that matrix: $\psi=\begin{pmatrix} A_1 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & A_G \end{pmatrix}-\begin{pmatrix} B_{11} & \cdots & B_{1G} \\ \vdots & & \vdots \\ B_{G1} & & B_{GG} \end{pmatrix}$, where A_g contains the term that depends on σ and $B_{g\tilde{g}}$ the rest. Notice that A_g is always square but $B_{g\tilde{g}}$ will be non-square when the number of products located in both segments is different. From this decomposition it is easier to obtain the inverse:

$$\{\psi^{-1}\}=\begin{cases} A_g^{-1}+\dfrac{A_g^{-1}B_{gg}A_g^{-1}}{1-V'A^{-1}U} & if\, g=\tilde{g} \\ \dfrac{A_g^{-1}B_{g\tilde{g}}A_{\tilde{g}}^{-1}}{1-V'A^{-1}U} & if\, g\neq\tilde{g} \end{cases} \qquad [A.2]$$

where each block corresponds with a segment $\forall\ g, \tilde{g} = 1, \dots, G$. The matrix $B_{g\tilde{g}}$ has been decomposed as: $B_{g\tilde{g}} = u_g v'_{\tilde{g}}$ where $u_g = \left(\alpha_1^g s_1^g, \dots, \alpha_{H_g}^g s_{H_g}^g \right)'$ and $v'_{\tilde{g}} = \left(s_1^{\tilde{g}}, \dots, s_{H_g}^{\tilde{g}} \right)$. The vectors that appear in this equation are $U = (u_1, \dots, u_G)$ and $V' = (v'_1, \dots, v'_G)$.

Finally, multiplying this inverse matrix by the vector of market shares I obtain the markup of product j belonging to segment g:

$$m_j = (1-\sigma)\left[\frac{1}{\alpha_j^g} + \frac{\sigma s_{j/g}}{1-\sigma \sum_{r=1}^{H_g} s_{r/g}} \frac{1}{s_j^g} \sum_{r=1}^{H_g} \frac{s_r^g}{\alpha_r^g} \right] + \frac{(1-\sigma)^2}{s_j^g\ L} \sum_{\tilde{g}=1}^{G} Z_j^{\tilde{g}} \left(\sum_{r=1}^{H_{\tilde{g}}} \frac{s_r^{\tilde{g}}}{\alpha_r^{\tilde{g}}} \right) \qquad \text{[A.3]}$$

where $L = 1 - (1-\sigma) \sum_{\tilde{g}=1}^{G} \left[\frac{\sum_{r=1}^{H_{\tilde{g}}} s_r^{\tilde{g}}}{1-\sigma \sum_{r=1}^{H_{\tilde{g}}} s_{r/\tilde{g}}} \right]$, and $Z_j^{\tilde{g}} = \left(s_j^g + \frac{\sigma s_{j/g} \sum_{r=1}^{H_g} s_r^g}{1-\sigma \sum_{r=1}^{H_g} s_{r/g}} \right) \frac{1}{1-\sigma \sum_{r=1}^{H_{\tilde{g}}} s_{r/\tilde{g}}}$.

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