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Location as an instrument for social welfare improvement in a spatial model of Cournot competition*

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Abstract:

Two-stage models are main frameworks in the analysis of oligopolistic competition. Literature has discussed some properties of such models when Cournot competition occurs in the second stage and assuming a non-spatial context. It finds that private and social optima are asymmetric. Using a spatial behavior with multiple marketplaces, the outcome is different. A social planner can use the location variable as an instrument for reallocating production from the equilibrium spatial pattern to the optimal outcome while maintaining the symmetry of the model.

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1. INTRODUCTION

In this work we will come across a much-debated question: Does the market mechanism allocate resources in an efficient way? The well-known First Theorem of Welfare Economics answers this question by saying that perfect competition achieves an efficient allocation of resources. However, this theorem can be easily adapted to demonstrate that Cournot models never do achieve that efficient solution. This fact has motivated the analysis of welfare properties of imperfect competition by several authors.[1]

Here, we try to contribute a little more to this topic by studying welfare properties of location models. Oligopoly models where firms take prior actions which later affect their marginal costs have long been analyzed by economic literature. Some researchers within this literature have discussed an interesting property in the theory of Cournot equilibria that arises because of the way aggregate production costs change in response to changes in the cost structure of the industry.

This property says that, in Cournot games where firms produce at constant marginal costs, the industry output and the industry price that solve the first order conditions for a Cournot-Nash equilibrium depend only on the sum of the firms' marginal costs, and not on their distribution across the firms, provided the Cournot equilibrium is interior. This property has been studied, among others, by Dixit and Stern (1982), Katz (1984) or Bergstrom and Varian (1985a). An immediate implication of this is that increases in social welfare (consumer surplus plus producer profits) will result if and only if the

[1] Luis Corchón (1996, p. 61) makes an extensive analysis of welfare and Cournot competition, showing the main results within this topic.

change in the vector of marginal costs induces a reduction in aggregate production costs.

Bergstrom and Varian (1985b) go more deeply into the above property and show that aggregate production costs are a decreasing function of the variance of the marginal production costs across the firms in the economy. This implies that aggregate production costs will be maximized whenever every firm has the same marginal cost. So, a social planner will then try to maximize the difference between firms' marginal costs in order to increase the social welfare in the economy.

As Salant and Shaffer (1999) point out, given Bergstrom and Varian's results and disregarding equity considerations, the asymmetry in Cournot models could then have both social and private advantages. Moreover, they also conclude that above properties have important implications for two-stage models. It is well known that identical firms playing a Cournot game, as it is commonly considered by economists, after simultaneously taking actions in the first stage that affect the second-stage, marginal costs generally give rise to identical actions in the first stage. However, by taking into account Bergstrom and Varian (1985b)'s conclusion, an asymmetric behavior in the first stage is required to maximize social welfare, regardless of whether it is costless to engage in. This implies that the symmetric restriction considered in those models and used for the reason of simplifying the analysis may be non-innocuous. [2]

In this circumstance, a social planner who cares about the social welfare of economic agents may be interested in subsidizing some firms in order to impose an asymmetry in the market. The government may be then forced to help some firms to the detriment of

[2] Salant and Shaffer (1999) analyze two set of examples of two-stage models with Cournot competition at the second stage where asymmetry can be introduced in the first period. In the first set of examples, asymmetries are introduced without cost and, in the second set, asymmetries in marginal costs are costly to introduce.

other firms, generating an equity problem that may be misunderstood by them. For example, as Salant and Shaffer (1999) point out, in the context of a learning-by-doing model, governments are often pressed to help their own domestic firms to move down the learning curve ahead of foreign competitors so that domestic firms can operate at a cost advantage in the future.

The above analysis of Bergstrom and Varian (1985a,b) and Salant and Shaffer (1999) is examined in the context of a traditional non-spatial economy in which distance costs are insignificant and negligible. However, the single market approach does not fit very well with some facts. According to location theory, firms' sales mainly occur between several cities with different locations in the space. Also, according to location theory, firms must decide their plants' location from which to serve demand. This implies that a location decision actually carried out by a firm which tries to move close to a given city will may affect the social welfare in all the cities involved.

In a spatial world, the assumption of a single market must be then relaxed, however, insofar as firms choose locations and serve a product to multiple market places. Moreover, considering multiple marketplaces contributes new and interesting properties to the welfare analysis, that, to our knowledge, have not yet been studied. Contrary to the single market approach, although we consider a symmetric behavior with identical firms, we show that social welfare can be improved by a social planner avoiding any equity problem. Firms obtain identical profits at the optimal locations and markets have the same level of social welfare. Thus, this result is sharply different from that derived in non-spatial economies.

More precisely, we consider a two-stage game where Cournot-type duopolists discriminate over two marketplaces. Consumers are then found in concentration at two discrete locations, such as cities joined by a major highway. Spain, and indeed much of the world, abounds with examples of such city-pairs. Examples include Barcelona/Valencia and Madrid/Bilbao. Moreover, firms locate at a site in between both firms and serve both cities from it.

In the first stage of the game, firms decide their locations in the market non-cooperatively and Cournot competition occurs in the second stage of the game. We focus on the case where firms do not incur any production costs and the marginal transportation costs are constant (see, for example, the solution of this location problem in Hwang and Mai, 1990,[3] or Anderson and Neven, 1991[4]). We also consider the minimal assumptions on the inverse demand function and marginal transportation costs to secure the existence of an interior solution, avoiding any specific functional form for them.

This paper demonstrates that unlike the case of single market context, if some type of coordination is possible at the first stage (a social planner, for example), the economy can achieve its social and private optimum while maintaining the symmetry of the model. We also show that the principle of maximum differentiation prevails in the optimum solution.

The paper is organized in the following way. Section 2 presents the model. Section 3 analyzes welfare improvement for single-market models. Section 4 generalizes previous results for the spatial model. Section 5, analyzes welfare improvement in a two-marketplace model. Finally, Section 6 concludes the paper.

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[3] Hwang and Mai (1990) study the effects of spatial price discrimination on output, welfare, and location of a monopolist in a two-market economy. They conclude that with linear demand curves the monopolist locates in one of the two markets and that total output under discriminatory pricing is less than that under mill pricing.

[4] Anderson and Neven (1991) solved the location game by considering two firms and consumers uniformly distributed on a segment. The authors showed that, for linear demand and convex transport costs, competition among firms leads to spatial agglomeration.

2. The model

Following classical models in the theory of oligopoly, we consider a duopolistic sector producing a homogeneous good and a competitive sector producing a composite good which is taken as the *numeraire*.

We consider two firms that locate on a line (main road) between two different markets (cities) denoted by *A* and *B*, each of which is located at a point. Each firm serves both markets by shipping the homogeneous good without fear of resale between markets. The markets are s miles apart and are connected by a main road as shown in Figure 1. Market *A* is located at point $x = 0$ and market *B* is located at point $x = s$.

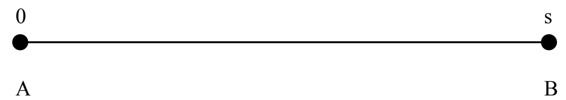

Figure 1. Spatial market

We assume symmetric constant-return-to-scale technologies, i.e., both firms produce at same constant marginal costs. Then, without loss of generality, the marginal cost can be set to zero. Let x_1 and x_2 denote the location of firm 1 and 2, respectively, for $x_i \in [0, s]$, $i = 1, 2$. Then, firm *i* pays a transportation cost tx_i (respectively, $t(s - x_i)$) to ship a unit of the homogeneous good from its own location to market *A* (respectively, market *B*), for $t > 0$.

We will analyze a two-stage non-cooperative game with location choice at the first stage and Cournot competition at the second stage. As usual, we solve the model by backward induction so that we first characterize the equilibrium in the second stage for given locations.

Since marginal production costs are constant and arbitrage is nonbinding, quantities set at different markets by the same firm are strategically independent (*no arbitrage*). The second stage Cournot equilibrium can then be characterized by a set of independent Cournot quantities, one for each of the two markets. The structure of the general location game used in this paper with Cournot duopolists who discriminate over space was firstly solved by Anderson and Neven (1991).

Let us first calculate the Cournot quantities in the second stage, assuming the firms' locations are given. At each market, inverse demand is given by $P_j = P\left(Q_j\right)$, where P_j is the market price and $Q_j = q_{1j} + q_{2j}$ is industry output in the market, for $j = A, B$. Under these assumptions, firm i's profits at market *A* and *B* are $\pi_{iA} = P\left(Q_A\right) q_{iA} - t x_i q_{iA}$ and $\pi_{iB} = P\left(Q_B\right) q_{iB} - t\left(s - x_i\right) q_{iB}$, respectively.

Before we proceed further, we make two assumptions about demand.

Assumption 1: $P'\left(Q_j\right) < 0$, for $j = A, B$.

Assumption 2: $2P'\left(Q_j\right) + P''\left(Q_j\right) Q_j < 0$, for $j = A, B$.

Assumption 1 requires simply that the inverse demand is downward sloping. Assumption 2 is equivalent to assuming diminishing marginal revenues, which ensures downward sloping best reaction function and guarantees the existence of equilibrium in Cournot competition (Novshek, 1985).

If we also assume that the Cournot equilibrium is an interior solution, this is a solution where firms produce strictly positive output (i.e., $q_{ij} > 0$ for $i = 1, 2$ and $j = A, B$).[5] Then, Cournot equilibrium at each market is determined by the following first-order conditions:

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[5] In the linear model we must assume that $P(0)$ is sufficiently large to ensure an interior solution.

$$P(Q_A)+P'(Q_A)q_{iA}-tx_i=0, \tag{1}$$

and

$$P(Q_B)+P'(Q_B)q_{iB}-t(s-x_i)=0. \tag{2}$$

Solving equations (1) and (2), we have:

$$q_{iA}=\frac{tx_i-P(Q_A)}{P'(Q_A)}, \tag{3}$$

and

$$q_{iB}=\frac{t(s-x_i)-P(Q_B)}{P'(Q_B)}. \tag{4}$$

From the above expressions, it is clear that q_{ij} is independent of both i and j when firms locate at the midpoint between both marketplaces, that is, $x_1=x_2=s/2$.

Before solving the first stage of the game, in the next section we will analyze a property of Cournot models that, as we will show, has interesting implications for the social welfare of each market.

3. The welfare in a single market

It is well known (see, for example, Dixit and Stern, 1982, Katz, 1984, or Bergstrom and Varian, 1985a) that in a single market model where n firms with constant marginal costs play a Cournot-Nash game, if the vector of constant marginal costs at a given market is changed exogenously without altering the sum of its components, the industry output (Q) will not change. This result is shown in the following proposition, (see Salant and Shaffer (1999) for an accurate analysis of this result and its implications for social welfare).

PROPOSITION 1: *Suppose the constant marginal costs of* n *firms in an industry are rearranged in a way which preserves their sum and results in a new Cournot-Nash equilibrium which is also interior. Then, industry output will be unchanged.*

An implication that can be derived from Proposition 1 is that the social welfare (the sum of consumer surplus plus industry output) in the market will solely depend on the change produced in the aggregate costs, since total revenue and gross consumer surplus are unaffected. This is so because both industry revenue ($QP(Q)$) and gross consumer surplus ($\int_0^Q P(u)\,du$) depend only on industry output (Q).

Next, we apply Proposition 1 to market *A* (the same applies to market *B*). Summing up equation (1) for all i yields:

$$2P(Q_A)+P'(Q_A)Q_A=\sum_{i=1}^{2} tx_i\ . \tag{5}$$

Equation (5) defines the price and the industry output in the Cournot equilibrium at market *A*. From the above equation, it is clear that equilibrium industry output (Q_A) only depends on the sum of the marginal transportation costs (tx_1+tx_2) and not on the distribution of those costs between firms, assuming that we obtain an interior solution. Therefore, an implication of this result is that the social welfare at market *A* will depend only on aggregate transportation costs while the sum of marginal transportation costs remains constant.

Find $P'(Q_A)$ in equation (5) and substitute in equation (3). Thus, the output of each firm at market *A* can be written as a proportion of the industry output Q_A, that is,

$$q_{iA}=\left[\frac{P(Q_A)-tx_i}{\big(P(Q_A)-tx_1\big)+\big(P(Q_A)-tx_2\big)}\right]Q_A\ . \tag{6}$$

From equation (5), we can also derive the industry profit at market *A* which is given by the expression:

$$\Pi_A = \pi_{1A} + \pi_{2A} = \frac{1}{2} Q_A \sum_{i=1}^{2} t x_i - \frac{1}{2} Q_A^2 P'(Q_A) - \sum_{i=1}^{2} t x_i q_{iA} \tag{7}$$

(see Appendix A) where q_{iA} is the equilibrium output of equation (6).

Bergstrom and Varian (1985b) also demonstrate that aggregate costs are maximized when every firm has the same marginal cost, concluding that the asymmetry between the cost structures of firms have social advantages. This property is shown in the following proposition.

PROPOSITION 2: *Industry production cost is a decreasing function of the variance of the marginal production costs across firms in the industry, assuming an interior Cournot equilibrium.*

The above result is demonstrated in Bergstrom and Varian (1985b). Thus, a social planner constrained to intervene in market *A* (without taking into account market *B*) could then improve social welfare just by allocating firms at different distances from the marketplace *A*, i.e., $x_1 \neq x_2$.

Propositions 1 and 2 also apply at marketplace *B*. Thus, equilibrium industry output (Q_B) only depends on the sum of the marginal transportation costs at market *B* ($t(s-x_1)+t(s-x_2)$), assuming we are in an interior equilibrium. Given equations (4) and (5), the output of each firm at market *B* is a proportion of the industry output Q_B, that is,

$$q_{iB} = \left[\frac{P(Q_B) - t(s-x_i)}{\left(P(Q_B) - t(s-x_1)\right) + \left(P(Q_B) - t(s-x_2)\right)} \right] Q_B, \tag{8}$$

and industry profit in market *B* is then given by:

$$\Pi_B = \pi_{1B} + \pi_{2B} = \frac{1}{2} Q_B \sum_{i=1}^{2} t(s-x_i) - \frac{1}{2} Q_B^2 P'(Q_B) - \sum_{i=1}^{2} t(s-x_i) q_{iB}. \tag{9}$$

The above properties of Cournot models are examined in the context of a single-market economy. Apparently, it is important to see whether this result can be applied to a spatial economy with multiple markets. By considering a spatial economy with multiple marketplaces (for example, two cities), a new element, the location variable, is incorporated in the analysis, which may be used by a social planner who maximizes the global welfare of the economy.

In the next section we study the implications of Propositions 1 and 2 in a spatial context with two separated markets.

4. The welfare in the spatial model

Let us consider now two marketplaces (two cities, for example) for given firms' locations x_1 and x_2. The global welfare of the economy is then the sum of the global consumer surplus ($\int_0^{Q_A} P(u)\,du + \int_0^{Q_B} P(u)\,du$) plus the global profits ($\Pi_A + \Pi_B$).

A reallocation of any firm (say firm 1, $x_1 + \Delta x$) will affect as much the sum of the marginal transportation costs at market A ($tx_1 + tx_2$) as the sum of the marginal transportation costs at market B ($t(s - x_1) + t(s - x_2)$). So, the property of invariable industry output at both market (Q_A and Q_B) is maintained so long as the change in the location does not simultaneously modify the values of $tx_1 + tx_2$ and $t(s - x_1) + t(s - x_2)$. That is, $tx_1 + tx_2 = t(x_1 + \Delta x) + tx_2$ and $t(s - x_1) + t(s - x_1) = t(s - x_1 - \Delta x) + t(s - x_2)$. In this case, the global welfare in the economy will depend only on the sum of the aggregate production costs at markets A and B ($\sum_{i=1}^{2} tx_i q_{iA} + \sum_{i=1}^{2} t(s - x_i) q_{iB}$), where q_{iA} and q_{iB} are the Cournot equilibria of equations (6) and (8).

Given the characterization of the equilibrium in the second stage, we now consider the duopoly's localization decisions in the first stage. In the first stage, firm i chooses its location, $x_i \in [0, s]$, to maximize its profit, taking as given the location of the other firm. Let us denote by x_1^* and x_2^* the Nash equilibrium locations, assumed to exist. For example, assuming linear demands, ($P(Q_j) = \alpha - \beta Q_j$, for $j = A, B$), there is a unique Nash equilibrium location where firms locate at the center of the market, ($x_1^* = x_2^* = s/2$).

Given that the marginal transportation costs incurred by the firms to ship the output to both markets depend on the location of them, the Nash equilibrium location will determine then the level of welfare at the second stage of the game. Let's turn now to the problem of whether or not this level of social welfare at the subgame perfect equilibrium can be improved while maintaining the global output constant. In the next section, we demonstrate that this is possible just by using the location variable and maintaining the symmetry of the model.

5. The welfare at the subgame perfect equilibrium

From equations (7) and (9), we can compute the global equilibrium profits. These profits are given by the following expression:

$$\begin{aligned}\Pi^* = {} & \frac{1}{2} Q_A^* \sum_{i=1}^{2} t x_i^* - \frac{1}{2}\left(Q_A^*\right)^2 P'\left(Q_A^*\right) + \frac{1}{2} Q_B^* \sum_{i=1}^{2} t\left(s - x_i^*\right) - \frac{1}{2}\left(Q_B^*\right)^2 P'\left(Q_B^*\right) \\ & - \sum_{i=1}^{2} t x_i^* q_{iA} - \sum_{i=1}^{2} t\left(s - x_i^*\right) q_{iB}.\end{aligned} \tag{10}$$

Moreover, we can also compute the global equilibrium consumer surplus. This equilibrium surplus is given by the expression:

$$CS^* = \int_0^{Q_A^*} P(u)\,du + \int_0^{Q_B^*} P(u)\,du \,. \tag{11}$$

From the equilibrium global profits and the consumer surplus, we then obtain the equilibrium global social welfare of the economy which is given by $W^* = CS^* + \prod^*$.

Our interest is two-fold. First, we ask ourselves whether or not the global equilibrium welfare is also optimal from the viewpoint of social welfare. Second, if it is not, whether or not the welfare improvement can be achieved while maintaining the symmetry of the model and without discriminating between the firms. As we mentioned in the introduction to the paper, the optimal solution in the single-market model implies that firms obtain different profits.

We will focus our analysis on studying symmetric Nash locations. These are location patterns such that:

Assumption 3: $tx_1^* = t\left(s - x_2^*\right)$ and $t\left(s - x_1^*\right) = tx_2^*$.

The above assumption would imply that $x_1^* + x_2^* = s$ (see Figure 2).

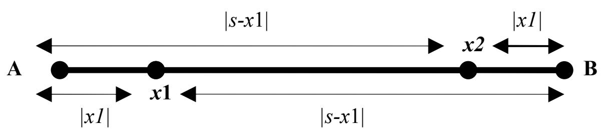

Figure 2. Symmetric locations with linear transportation costs.

Under Assumption 3, the sum of marginal transportation costs at both marketplaces satisfies:

$$tx_1^* + tx_2^* = t\left(s - x_1^*\right) + t\left(s - x_2^*\right) = ts \,. \tag{12}$$

We cannot exclude the possibility of the existence of asymmetric Nash locations. However, given the symmetry of the model, asymmetric equilibriums may be improbable. We assume, without loss of generality, that $x_1^* \le x_2^*$, i.e., firm 1 is closer to marketplace *A* than firm 2, whereas firm 2 is closer to marketplace *B* than firm 1 (see Figure 2). Thus, the global profit can be written as:

$$\Pi^* = \left(Q^*\right)^2 \left|P'\left(Q^*\right)\right| + \frac{t^2\left(s-2x_1^*\right)^2}{\left|P'\left(Q^*\right)\right|}, \tag{13}$$

where $Q^* = Q_A^* = Q_B^*$ (see Appendix B). In equation (13), the impact on the global profits of reallocating firms, while maintaining the sum of marginal transportation costs at both marketplaces, depends only on the term $\left(s-2x_1^*\right)$. This term represents the level of concentration between firms ($s-2x_1^* = x_2^* - x_1^*$), which is positive since we have assumed that $x_1^* \leq x_2^*$. Thus, the global profit is maximized when $x_1^* = 0$ and $x_2^* = s$, that is, when a single firm is located in each market. The following proposition summarizes the previous result:

PROPOSITION 3: *Given symmetric Nash equilibrium locations,* x_1^* *and* x_2^**, suppose that the locations of both firms in the economy are rearranged in a symmetrical way,* x_1 *and* x_2*, which* $s = x_1 + x_2$*. Then, if* $x_1 < x_1^*$*, the global welfare and the global profits increase after the rearrangement (provided the induced Cournot equilibrium is interior).*

Proposition 3 states a sufficient condition for social welfare improvement in the spatial economy. It says that all symmetric Nash solutions of the two-stage game where firms 1 and 2 do not locate at marketplaces *A* and *B*, respectively, are not optimal solutions from the point of view of a social planner who maximizes the welfare of the economy.

In the linear model, for example, the only Nash equilibrium locations are $x_1^* = s/2$ and $x_2^* = s/2$. That is, firms concentrate at the midpoint between both marketplaces (*principle of minimum differentiation*). In this equilibrium configuration, the global welfare is not maximized, and a social planner can increase it by dispersing firms and locating them close to the markets.

The intuition behind this result is as follows. In the spatial model analyzed here two opposing forces are in play: imperfect competition and transportation costs. On the one hand, imperfect competition results in dispersing firms in order to reduce competition. On the other hand, transportation costs produce concentration of production at the midpoint between both markets, to reduce thereby, the distance to the marketplaces.

Thus, in a competitive framework and assuming a linear market (linear demand and marginal transportation costs), it is obtained that reduction in transportation costs to the marketplaces acquires greater significance than decreasing competition. However, the opposite occurs when a social planner tries to maximize the social welfare of the economy. The weakness of quantity competition by dispersing production is optimal from the point of view of social welfare.

COROLLARY 1: *Suppose a symmetric Nash equilibrium location where* $x_1^* = 0$ *and* $x_2^* = s$*. Global welfare and global profits cannot be increased by relocating both firms in a way which results in a different symmetric configuration, (provided the induced Nash output is interior).*

Corollary 1 concludes that, among all symmetric spatial configurations, the global welfare and the global profits of the economy are maximized at the spatial configuration where a single firm is located in each market, that is, $x_1 = 0$ and $x_2 = s$ (*principle of maximal differentiation*), contrary to the Nash competitive solution.

Next, we compute the welfare gain experienced by the economy when a social planner reallocates firms from a symmetric Nash equilibrium configuration, $x_1^* \neq 0$ and

$x_2^* \neq s$, to the optimal spatial configuration, $x_1 = 0$ and $x_2 = s$, assuming that at both spatial configurations the corresponding Cournot equilibria are interior.

The first assumption requires simply that for these spatial configurations the Cournot game has an interior solution. The second assumption states that the condition for the sum of the marginal transportation costs is constant at both marketplaces after the rearrangement. This implies that although there is a relocation of production, industry equilibrium outputs Q_A^* and Q_B^* will remain constant.

The change produced in the social welfare with the reallocation of both firms is two-fold. First, firm 1 moves its production from location x_1^* to the marketplace *A*, generating a cost saving, and, at the same time, firm 2 moves away from market *A*, so that this firm loses a share of its sales in market *A* to the benefit of firm 1, also making a cost saving. Moreover, since locations are symmetrical, firm 1 increases its sales in market *A* to the same extent that firm 2 reduces its sales in this market. Second, the market share that firm 2 maintains in market *A* must be shipped from market *B*, generating a cost increase.

All these cost savings and losses in the transportation costs define the welfare increase in the economy after the reallocation. Given equation (13) and for $x_1^0 = 0$ and $x_2^0 = s$, this welfare increase can be written as:

$$\Delta W\left(x_1^*\right) = W^0 - W^* = \frac{4t^2 x_1^*\left(s - x_1^*\right)}{\left|P'\left(Q^*\right)\right|}. \tag{14}$$

With a linear demand ($P(Q) = \alpha - \beta Q$), $\Delta W = t^2 s^2 / \beta$. By differentiating ΔW with respect to t, $\frac{\partial \Delta W}{\partial t} = 2ts^2 / \beta > 0$. For arbitrary values of the parameters this derivative is always positive. Thus, decreasing marginal transportation costs will reduce the benefits of reallocating firms from the Nash solution (minimal differentiation) to the optimal solution (maximal differentiation). By differentiating ΔW with respect to s, $\frac{\partial \Delta W}{\partial t} = 2t^2 s / \beta > 0$. For arbitrary values of the parameters this derivative is also positive. Thus, closer cities also reduce the benefits of reallocation.

An implication of this is that in a situation where marketplaces are very close, an economic policy of reallocation and infrastructure improvement may result in a low benefit after the reallocation. If, in addition, there are some reallocation costs, it is possible that the change will not take place.[6]

We now turn our attention to the problem of how to drive equilibrium locations to the optimal locations while maintaining the level of global output and the symmetry of the model.

PROPOSITION 4: *Given a symmetric subgame perfect equilibrium where* $x_1^* \neq 0$ *and* $x_2^* \neq s$, *a net benefit can be achieved using location as an instrument and with an equal treatment of firms.*

This result can be compared with that obtained in the single-market model. In a wide variety of two-stage games with a single market (see, for example, Funderberg and Tirole, 1983), since firms are ex ante identical, the subgame-perfect equilibrium will be symmetrical. A social planner constrained to intervene in the first period could improve the welfare on/of the market by imposing different marginal production costs on the firms. Thus, to intervene optimally, an unequal treatment of identical firms would be required, with the subsequent equity problem.

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[6] The consideration of reallocation costs is outside the scope of this paper. This problem has been analyzed by Salant and Shaffer (1999) in a spaceless context.

Proposition 4 shows that in a spatial model with two marketplaces, the social planner can increase the global profit and the global welfare of the economy by reallocating the firms and without discriminating between firms, just by locating each firm at a different marketplace.

From the point of view of each market, firms' marginal transportation costs are different ($tx_1 \neq tx_2$ and $t(s-x_1) \neq t(s-x_2)$) after the reallocation, and it is in this context that we cannot say firms are symmetric. However, from the point of view of the global economy, we can say that firms are symmetrical since by interchanging firms' location, firms' profits do not change.

6. Conclusions

Using a two-stage game where Cournot-type duopolists discriminate over two marketplaces, this paper demonstrates that unlike the case of a single market context, if some type of coordination is possible at the first stage (a social planner, for example), the economy can achieve the social and private optimum while maintaining the symmetry of the model. That is, both firms obtain the same profits at the optimal locations and both marketplaces have the same level of social welfare.

Salant and Shaffer (1999) show, in a single market model, that it is sometimes both socially and privately optimal to invest asymmetrically in the prior stage. Moreover, authors point out that unlike the case of a fully decentralized economy where a symmetric context gives rise to a symmetric investment in the prior stage, in a centralized economy, asymmetric investments would be the norm.

In contrast, in a spatial behavior with two market places, if the economy is fully centralized with a social planner controlling economic variables, we have a new instrument that can be used for achieving the desired optimal solution. This is the location of firms. Therefore, if the welfare-maximizer social planner is constrained to maintain symmetry in the economy so that it does not discriminate among economic agents, the social planner can reallocate firms in such a way that this symmetry is maintained.

We also show that the principle of maximum differentiation prevails in the optimum solution. In the spatial model analyzed here two opposing forces are in play: imperfect competition and transportation costs. On the one hand, imperfect competition results in dispersing firms in order to reduce competition. On the other hand, transportation costs produce concentration of production at the midpoint between both markets to reduce thereby the distance to the marketplaces. Thus, we obtain that in a centralized framework, the weakness of quantity competition by dispersing production is optimal from the point of view of social welfare. By contrast, in a decentralized model (assuming a linear inverse demand function), firms agglomerate (principle of minimum differentiation), indicating that the competitive solution is far from the optimum.

We show that with a linear demand, in a situation where two cities are very close, an economic policy of reallocation and infrastructure improvement may result in a low benefit after the reallocation. If, in addition, there are some reallocation costs, it is possible that the change will not take place. However, this possibility has not been considered in this paper.

APPENDIX A

In order to simplify notation, we drop the subscript A. Multiply both sides of equation (5) by Q,

$$2P(Q)Q + P'(Q)Q^2 = Q\sum_{i=1}^{2} tx_i .$$

Add $-2\sum_{i=1}^{2} tx_i q_i$ at both sides of the above equation,

$$2P(Q)Q - 2\sum_{i=1}^{2} tx_i q_i + Q^2 P'(Q) = Q\sum_{i=1}^{2} tx_i - 2\sum_{i=1}^{2} tx_i q_i .$$

Thus, since $\Pi = P(Q)Q - \sum_{i=1}^{2} tx_i q_i$, we obtain

$$\Pi = \frac{1}{2}Q\sum_{i=1}^{2} tx_i - \frac{1}{2}Q^2 P'(Q) - \sum_{i=1}^{2} tx_i q_i . \text{ Q.E.D.}$$

APPENDIX B

Given that $x_1^* + x_2^* = s$, we can write the global transportation costs as:

$$C^* = \sum_{i=1}^{2} tx_i^* q_{iA}^* + \sum_{i=1}^{2} t\left(s - x_i^*\right) q_{iB}^* .$$

Rearranging the above expression, we obtain:

$$C^* = tsQ_B^* + \sum_{i=1}^{2} tx_i^* \left(q_{iA}^* - q_{iB}^*\right).$$

Given the symmetry of the model, $q_{1A}^* = q_{2B}^*$ and $q_{1B}^* = q_{2A}^*$, then using these equalities, we can express the global transportation costs as:

$$C^* = tsQ_B^* + tx_1^*\left(q_{1A}^* - q_{2A}^*\right) + tx_2^*\left(q_{2A}^* - q_{1A}^*\right).$$

Since $x_1^* + x_2^* = s$, we obtain:

$$C^* = tsQ_B^* - t\left(s - 2x_1^*\right)\left(q_{1A}^* - q_{2A}^*\right).$$

By symmetry, $Q_A^* = Q_B^*$. From now on, we will denote $Q^* = Q_A^* = Q_B^*$. We can then express the global profit in equation (10) as a function of the variables in market *A*:

$$\Pi^* = \frac{1}{2}tsQ^* - \frac{1}{2}\left(Q^*\right)^2 P'\left(Q^*\right) + \frac{1}{2}tsQ^* - \frac{1}{2}\left(Q^*\right)^2 P'\left(Q^*\right) - tsQ^* + t\left(s - 2x_1^*\right)\left(q_{1A}^* - q_{2A}^*\right),$$

which yields:

$$\Pi^* = \left(Q^*\right)^2 \left|P'\left(Q^*\right)\right| + t\left(s - 2x_1^*\right)\left(q_{1A}^* - q_{2A}^*\right).$$

Using equation (6) for $i = 1, 2$ yields

$$q_{1A}^* - q_{2A}^* = \frac{t\left(s - 2x_1^*\right)Q^*}{2P\left(Q^*\right) - ts}.$$

From equation (5), we obtain

$$q_{1A}^* - q_{2A}^* = \frac{t\left(s - 2x_1^*\right)}{\left|P'\left(Q^*\right)\right|}.$$

By substituting the above expression in $\prod^*$ we conclude the demonstration. Q.E.D.

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