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What are we assuming when using inequality measures to quantify geographic concentration? An axiomatic approach*

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Abstract

This paper formally shows the parallel that exists between inequality and spatial concentration measurement. This examination allows us to unveil the properties that the literature is implicitly assuming when using inequality measures to quantify the spatial concentration of economic activity. Thus, the properties satisfied by the Gini index and the generalized entropy family when using them to analyze location patterns are shown. In addition, another inequality-based concentration measure is proposed. Finally, the basic properties of the concentration measurement when using "employment Lorenz curves" are unveiled, and additive decompositions of these curves are proposed.

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1. Introduction

In recent years, the study of spatial concentration patterns of economic activity has received increasing interest in the field, both empirically and theoretically. This flourishing is in part motivated by general concern about the effects of economic integration processes on production location patterns, especially in Europe where the creation of the Single Market has stimulated the debate (Amiti, 1999; Haland et al., 1999; Brülhart, 2001; and Aiginger and Pfaffermayr (2004), inter alia).[1]

Geographic concentration measures quantify the extent to which the distribution of a sector across space departs from a no-concentration benchmark (see Brülhart and Traeger, 2005; Brakman et al., 2005). In particular, according to *absolute* concentration measures, the no-concentration benchmark is that where the sector has a uniform distribution across locations. Regarding *topographic* concentration measures, no-concentration exists instead if the sector is evenly spread over physical space, for example, per square kilometer. However, most concentration measures have followed a different approach. Suppose, for example, that economic activity is measured in terms of employment, as traditionally assumed, and the focus is on manufacturing industries. In this case, the distribution of overall manufacturing employment is usually considered the distribution of reference against which to compare that of any single sector, so that no spatial concentration exists in the sector so long as its employment distribution across locations is equal to that of the industrial aggregate. This notion is labeled *relative* concentration and has been extensively used in empirical research.

Among the spatial concentration measures existing in the literature, those borrowed from the literature on income inequality are some of the most widely used.[2] In this regard, the Gini index has been traditionally used for analyzing the spatial location patterns of manufacturing industries (Krugman, 1991; Amiti, 1999; Brülhart, 2001, Suedekum, 2006, among many others), while lately some of the indexes included in the generalized entropy family have been used as well (Brülhart and Traeger, 2005; Brakman et al. 2005). These inequality-based

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[1] From a theoretical perspective, the literature of the new economic geography has contributed extensively to this debate. A review of this literature can be seen in Fujita *et al*. (2000), Neary (2001), and Ottaviano and Thisse (2004), among others.

[2] Other concentration measures proposed in the literature are formally derived from location models (Ellison and Glaeser 1997; Maurel and Sédillot 1999; Guimarães et al., 2007). There are also distance-based measures related to the literature on spatial statistics (Marcon and Puech, 2003; Duranton and Overman, 2005).

concentration measures have been recently analyzed by Bickenbach and Bode (2006), who proposed a taxonomy of concentration measures based on three elements: the distribution of reference against which comparing the distribution of the sector of study; the weight of each location; and the projection function, which measures how much the sector departs from the benchmark in each location, and aggregates such differences taking into account the weighting scheme. These authors suggest that even though the distribution of reference is traditionally used to obtain the weighting scheme, this does not necessarily have to be the case, since both elements can be chosen separately, which opens the door to new concentration measures based on inequality measures. However, as far as we know, the implications of using indexes derived from the literature on income distribution to measure spatial concentration of economic activity have not yet been addressed.[3]

Certainly, in the context of income distribution, the properties of inequality measures are well known, since this literature has tackled inequality measurement from an axiomatic perspective (Shorrocks 1984; Foster, 1985; Cowell, 2000; Zheng, 2007). Consequently, there is general agreement with respect to the basic properties that any inequality measure should satisfy, even though there is not complete consensus regarding some other properties. This axiomatic approach has facilitated comparisons among inequality measures, and has permitted researchers to choose, in each empirical case, the index that is more suitable. The approach followed by the literature on geographic concentration has been rather different, since such an axiomatization does not exit (Combes and Overman, 2004). The aim of this paper is not to propose an axiomatic framework for measuring geographic concentration, but to unveil the properties we are implicitly assuming when using inequality measures in this context. As shown in this paper, the use of inequality-based concentration measures involves some axioms borrowed from the literature on income distribution, but also additional axioms that have recently been proposed in the literature on occupational segregation.

For this purpose, first of all, some basic axioms borrowed from the literature on income distribution and occupational segregation are adapted to our case (Foster, 1985; Hutchens, 1991, 2004; Alonso-Villar and Del Río, 2007). Second, by using them, the parallel between income inequality and geographic concentration measurement is formally established. Our

[3] Inequality measures have been studied in a spatial context by Shorrocks and Wan (2005) and Dawkins (2007), who analyzed decompositions of income inequality measures and income segregation, respectively, in space. Even though these studies combine inequality measures and space, none of them, however, tackled the topic of this paper.

analysis helps to explain the relationship that exists between the distribution of reference and the weighting scheme when using inequality indexes for regional studies. Third, it is shown that when choosing either the Gini index or any of the members of the generalized entropy family of indices to measure spatial concentration, the basic properties settled before are being implicitly assumed.[4] In addition, a new inequality-based concentration measure is proposed. Fourth, the basic properties behind the spatial concentration measurement when using "employment Lorenz curves" are revealed. Fifth, additive decompositions of this curve by subsectors and by groups of locations are proposed, because, even though additive decompositions of the generalized entropy family of indexes have been used to measure industrial concentration (Brülhart and Trager, 2005), as far as we know, no decompositions of the "employment Lorenz curves" have yet been suggested. One of the decompositions parallels that proposed by Bishop et al. (2003) in a context of income distribution, while the analog of the other can be seen in Alonso-Villar and Del Río (2008a) in a framework of occupational segregation.

The paper is structured as follows. Section 2 introduces a set of properties for measuring geographic concentration, and formally shows the parallel that exists between inequality measures satisfying some basic axioms and concentration measures satisfying the aforementioned properties. Section 3 introduces the "employment Lorenz curve" of a sector, and proposes two additive decompositions of this curve, one by subsectors and another by groups of locations. Finally, Section 4 draws some conclusions.

2. Geographic concentration: an axiomatic approach

In what follows, some basic axioms are established in order to measure geographic concentration. These properties are borrowed from the literature on income distribution and occupational segregation and are adapted to our case. These axioms will be used later to characterize some of the concentration indexes used in spatial analyses.

Consider an economy with $L > 1$ location units (counties, regions, countries, etc.) across which total employment, denoted by T, is distributed. Let $t \equiv (t_1, t_2, ..., t_L)$ denote this

[4] As opposed to what is usually mentioned in the literature, this paper also shows that the Gini index does not reach unity value when used for measuring geographical concentration.

distribution, where $T=\sum_l t_l$. This distribution represents the benchmark against which the distribution of any sector is compared (*relative* notion). If concerned, for example, with the geographic concentration of manufacturing industries, t could represent the distribution of manufacturing employment among regions (as in Amiti, 1999; and Brülhart, 2001). If concerned with a broader perspective, t could represent instead the distribution of overall employment, services included (as in Brülhart and Traeger, 2005).

Let us denote by $x\equiv\left(x_1,x_2,...,x_L\right)$ the employment distribution of the sector in which we are interested, and by X its employment level ($X=\sum_l x_l$). In this paper, an index of geographic concentration is a function $I_c:D\to\mathbb{R}$ such that $I_c(x;t)$ represents the concentration level of the sector having distribution x when comparing it with the distribution of reference t, where $D=\bigcup_{L>1}\left\{\left(x;t\right)\in\mathbb{R}^L{}_+\times\mathbb{R}^L{}_{++}:x_l\le t_l\forall l\right\}$.

In order to formally establish the relationship between the measurement of spatial concentration of economic activity and the measurement of income inequality, in what follows, a hypothetical "income" distribution derived from vector $\left(x;t\right)$ is obtained. In doing so, in each location l, x_l is allocated in equal amounts among t_l workers. In other words, in each location, the variable of study (employment in the sector of study) is equally split among all individuals (both those working in the sector of study and those in the remaining sectors). This per capita employment level, $\frac{x_l}{t_l}$, represents the employment in the sector of study that corresponds to each individual in location l, and it plays the role of individual "income." Namely, the fictitious "income" distribution is constructed as follows: there are t_1 persons with an individual "income" of $\frac{x_1}{t_1}$, t_2 persons with an individual "income" of $\frac{x_2}{t_2}$, and so on. Therefore, we have built income distribution $y\equiv(\underbrace{\frac{x_1}{t_1},...,\frac{x_1}{t_1}}_{t_1\text{ individuals}},...,\underbrace{\frac{x_L}{t_L},...,\frac{x_L}{t_L}}_{t_L\text{ individuals}})$ in a world of $T=\sum_l t_l$ individuals where total "income" is $X=\sum_l t_l\frac{x_l}{t_l}$.

Suppose, for example, that we want to measure the geographical concentration of the chemicals sector by comparing its employment distribution across regions with that of manufacturing employment (in what follows, this example will be developed in order to make explanations easier). Consider that the economy has three locations and that the employment distribution of the chemicals industry among them is $(3,2,5)$, while the distribution of manufacturing workers is $(30,10,30)$. In other words, $(x;t)=(3,2,5;30,10,30)$. Therefore, our fictitious "income" distribution would be one with 70 people having a total income of 10 units: there are 30 people with an individual "income" of 0.1, 10 people with an individual "income" of 0.2, and 30 people with an individual "income" of 0.6, i.e., the "income" distribution is equal to $y \equiv \left(\underbrace{\frac{3}{30},...,\frac{3}{30}}_{30}, \underbrace{\frac{2}{10},...,\frac{2}{10}}_{10}, \underbrace{\frac{5}{30},...,\frac{5}{30}}_{30} \right)$.

The parallel between employment distribution $(x;t)$ and hypothetical "income" distribution y will be helpful for understanding the axiomatic framework presented in what follows.

2.1 Basic properties

There is a wide consensus in the literature on income distribution about the properties an inequality measure has to satisfy when it is used to compare income distributions having the same mean. Basically, one must invoke the symmetry axiom—which guarantees anonymity among individuals—and the Pigou-Dalton principle of transfers—which requires a transfer of income from a poorer to a richer person to increase inequality.[5]

We can start our list by adapting the first axiom to our context. We call this axiom *symmetry in locations*, which means that if locations are enumerated in a different order, the concentration index should remain unchanged.[6]

[5] Properties such as normalization, continuity, differentiability, and replication invariance are also commonly invoked, but they are of a more technical nature.
[6] In the occupational segregation literature, this axiom is called "symmetry in groups" and requires anonymity among occupations (see Hutchens, 1991).

Axiom 1: *Symmetry in locations*. If $\left(\Pi(1),...,\Pi(L)\right)$ represents a permutation of locations, then $I_c\left(x\Pi;t\Pi\right)=I_c\left(x;t\right)$, where $x\Pi=\left(x_{\Pi(1)},...,x_{\Pi(L)}\right)$ and $t\Pi=\left(t_{\Pi(1)},...,t_{\Pi(L)}\right)$.

As mentioned, another basic axiom of any inequality measure is the Pigou-Dalton principle. This property gives rise to our next axiom: *movement between locations.* If we focus again on the chemicals sector, this property requires that when a region with a lower employment level in chemicals than another (but with the same manufacturing employment) loses employment in chemicals in favor of the other location, concentration in the chemicals sector must increase.[7]

Axiom 2: *Movement between locations*. If $\left(x';t'\right)\in D$ is obtained from $\left(x;t\right)\in D$ in such a way that:

(i) location *i* loses employment in the sector of study, while the opposite happens to location *h*, i.e., $x'_i=x_i-d$, $x'_h=x_h+d$ $(0<d\le x_i)$, where *i* and *h* are two locations with the same aggregate employment level, $t_i=t_h$, but with different shares in the sector of study since $x_i<x_h$;

(ii) the employment level of the sector of study does not change in the remaining locations, i.e., $x'_l=x_l$ $\forall l\ne i,h$;

(iii) the employment share that each location represents in terms of the distribution of reference does not change, i.e., $\frac{t'_l}{T'}=\frac{t_l}{T}$ $\forall l$,

then $I_c\left(x';t'\right)>I_c\left(x;t\right)$.

In other words, if location *i* has initially the same manufacturing employment level as location *h*, but a lower employment level in chemicals, then a movement of employment in chemicals from location *i* to location *h* would be considered a disequalizing movement fostering the concentration of the sector.

[7] This axiom has also been adapted to measure occupational segregation, where it is called "movement between groups." Note that our definition is analogous to that proposed by Alonso-Villar and Del Río (2007) in a context of occupational segregation, but it differs from that of Hutchens (2004).

As mentioned above, the symmetry axiom and the Pigou-Dalton principle are the basic axioms required to compare income distributions having the same mean. However, if one is interested in comparing two income distributions that have different means, an additional property has to be specified, the one regarding the type of mean-invariance property. This requires introducing another judgment value into the analysis, and no agreement has been reached among scholars with respect to this matter. Some opt to invoke the scale invariance axiom, according to which the inequality of a distribution remains unaffected when all incomes increase (or decrease) by the same proportion. This property gives rise to “relative” inequality measures such as the well-known Gini index and the generalized entropy family of indexes, which are consistent with the Lorenz criterion. Others prefer, instead, to call on the translation invariance axiom, under which inequality remains unaltered if all incomes are augmented (or diminished) by the same amount, thereby giving rise to “absolute” inequality measures. The variance is an example of such a measure whose good properties have been discussed by Chakravarty (2001), among others.[8] It is important to note that the labels “relative” and “absolute” used in the literature on income distribution do not have the same meaning as labels *relative* and *absolute* in the literature on spatial concentration, whose definitions were mentioned in the introduction.

Since the “relative” inequality measures are the ones extended by scholars in the field of regional economics to measure the spatial concentration of economic activity, the next axiom included in our list is the *scale invariance* axiom, which is adapted to our context. More discussion on this matter will be given later when introducing the *translation invariance* axiom.

Axiom 3: *Scale Invariance*. If the distribution of reference, t, is multiplied by a positive scalar, a, and the distribution of the sector of study, x, is multiplied by another positive scalar, b, in such a way that $ax_l \leq bt_l$, then the concentration level of the sector does not change, i.e., $I_c\left(ax;bt\right)=I_c\left(x;t\right)$.

This property means that the value of the concentration index should not change when the employment level of the distribution of reference and/or that of the sector under consideration

[8] “Absolute” inequality measures are related to absolute Lorenz curves (Moyes, 1987) in a similar manner to the relationship that exists between “relative” inequality measures and traditional Lorenz curves.

vary, so long as the weight that each location represents in distributions t and x remains unaltered. In other words, if manufacturing employment in each location doubles and that of chemicals triples, provided that both facts are compatible, the spatial concentration level of chemicals should not change. Therefore, this axiom means that in measuring spatial concentration it is only employment shares that matter, not employment levels.[9]

Next, we present a property recently invoked in the literature on income distribution, unit consistency, which requires that inequality rankings between distributions do not change when all incomes are multiplied by a positive scalar (Zheng, 2007). In other words, it guarantees that inequality rankings are unaffected by the currency unit. In our context, this axiom requires that concentration rankings between employment distributions remain the same whether measuring employment in thousands or in hundreds of individuals.

Axiom 3': *Unit consistency*. Let $(x;t)$ and $(x';t')$ be two distributions such that $I_c(x;t) < I_c(x';t')$. A concentration measure I_c is labeled as unit consistent if $I_c(\theta x;\theta t) < I_c(\theta x';\theta t')$ for any $\theta \in \mathbb{R}_{++}$.

Certainly, any concentration measure that satisfies axiom 3 also verifies axiom 3', since the former axiom implies that $I_c\left(\theta x;\theta t\right) = I_c\left(x;t\right)$ for any $\theta \in \mathbb{R}_{++}$ and, therefore, if $I_c(x;t) < I_c(x';t')$, then $I_c(\theta x;\theta t) < I_c(\theta x';\theta t')$. Note that axiom 3 leads to measures that are cardinally unaffected by the unit of measurement, while axiom 3' leads instead to measures that are ordinally unaffected by the unit. This explains why there are inequality measures that are not scale invariant but satisfy, instead, this property. In fact, the variance is an "absolute" inequality measure that satisfies this axiom.

So far, we have proposed three axioms that are similar to those proposed in the literature on income distribution when using "relative" inequality indexes (symmetry, the Pigou-Dalton principle, and scale invariance). The necessity of the next axiom, *insensitivity to proportional subdivisions of locations,* does not arise, however, when working with income distributions. An income distribution is nothing but the distribution of an aggregate variable (total income) among individuals. However, in our context, the aggregate variable, chemicals employment,

[9] In a context of occupational segregation, a similar axiom has been proposed by Alonso-Villar and Del Río (2007).

is distributed among groups of individuals who share a common location, which requires an a priori classification of these location units according to a given spatial scale (counties, regions, states, etc.). This axiom requires that subdividing a location into several units of equal size, both in terms of aggregate employment and in terms of employment in the sector of study, does not affect the concentration level of the sector.[10] Without loss of generality, in the next axiom the subdivision is undertaken for the last location in order to make notation easier.

Axiom 4: *Insensitivity to proportional subdivisions of locations.* If $(x';t') \in D$ is obtained from $(x;t) \in D$ in such a way that:

(i) all locations except the last one remain unaltered both in terms of aggregate employment and employment in the sector of study, i.e., $t'_l = t_l$ and $x'_l = x_l$ for any $l = 1,...,L-1$;

(ii) the last location is subdivided in M location units without introducing any difference among them in terms of employment shares, i.e., $x'_l = x_L / M$, $t'_l = t_L / M$ for any $j = L,...,L+M-1$,

then, $I_c(x';t') = I_c(x;t)$.

In order to understand the relevance of the above axiom, we go back to the example given at the beginning of the section. Note that "income" distribution $y \equiv \left(\frac{3}{30},...,\frac{3}{30},\frac{2}{10},...,\frac{2}{10},\frac{5}{30},...,\frac{5}{30}\right)$ can be obtained from different $(x;t)$ vectors, depending on how the "income" data are grouped. We could, for example, group the "income" data as before, $\left(\underbrace{\frac{3}{30},...,\frac{3}{30}}_{group1(30)},\underbrace{\frac{2}{10},...,\frac{2}{10}}_{group2(10)},\underbrace{\frac{5}{30},...,\frac{5}{30}}_{group3(30)}\right)$, so that the 30 individuals having an "income" level equal to 0.1 are in group 1, the 10 individuals having an "income" of 0.2 are in group 2, and the 30 individuals having an "income" of 0.6 are in group 3. In this case, we would obtain former vector $(x;t) = (3,2,5;30,10,30)$. But we could also group individuals in five groups,

[10] In the case of segregation, the corresponding axiom is named "insensitivity to proportional divisions" (see Hutchens, 2004).

$$\left(\underbrace{\frac{3}{30},...,\frac{3}{30}}_{group1(10)},\underbrace{\frac{3}{30},...,\frac{3}{30}}_{group2(10)},\underbrace{\frac{3}{30},...,\frac{3}{30}}_{group3(10)},\underbrace{\frac{2}{10},...,\frac{2}{10}}_{group4(10)},\underbrace{\frac{5}{30},...,\frac{5}{30}}_{group5(30)}\right)$$, so that 10 of the individuals having an "income" of 0.1 are included in the first group, 10 are in group 2, and the remaining 10 are in the third group, while those having an "income" of 0.2 are included in the fourth group, and those with an "income" of 0.6 are in the fifth group. In this case, $(x';t') = (1,1,1,2,5;10,10,10,10,30)$. Note that, according to axiom 4, both $(x;t)$ and $(x';t')$ have the same concentration level since the latter can be obtained from the former by a proportional subdivision of locations.

Our next axiom, *aggregation*, is very helpful for empirical analyses since it has to do with the decomposition of indexes by subgroups. Like axiom 4, *aggregation* is related to spatial scale, but as opposed to it, location units are now aggregated rather than subdivided. In a later section, this axiom, together with axioms 1, 2, and 3, will allow us to characterize the generalized entropy family of indexes used to measure the geographic concentration of economic activity, whose members are additively decomposable.

Axiom 5: *Aggregation*. Let us assume that locations can be partitioned into two mutually exclusive groups so that $(x;t) = (x^1, x^2; t^1, t^2)$, where the aggregate employment level in locations included in group 1 (2) is denoted by T_1 (T_2), while X_1 (X_2) represents the employment level of the sector of study in the corresponding group of locations. Concentration index I_c is defined as aggregative if there exists a continuous aggregator function A such that $I_c(x,t) = A\left(I_c(x^1;t^1), \frac{X_1}{T_1}, T_1, I_c(x^2;t^2), \frac{X_2}{T_2}, T_2\right)$, where A is strictly increasing in the first and fourth argument.[11]

Therefore, the overall concentration level of the sector of study is a function of: (a) the concentration level of the sector in each group of locations (denoted by $I_c(x^1;t^1)$ in group 1);

[11] The formulation used here is analogous to that put forward by Hutchens (2004) to measure occupational segregation.

(b) the employment level in each group of locations (denoted by T_1 in group 1); and (c) the employment share of the sector in each group of locations (denoted by $\frac{X_1}{T_1}$ in group 1).

2.2 Relationship between spatial concentration and inequality measurement

In what follows, we first show the parallel that exists between concentration indexes satisfying the aforementioned axioms and inequality indexes satisfying some basic properties.[12] Second, concentration measures derived from "relative" inequality measures and satisfying the above axioms are characterized. Third, a new concentration index derived from an "absolute" inequality measure is proposed.

Proposition 1. *An index of geographical concentration* I_c *satisfying axioms 1, 2, 3, and 4 can be regarded as an inequality index* I *satisfying symmetry, the Pigou-Dalton transfer principle, replication invariance, and scale invariance.*[13] *Namely, the concentration index evaluated at* $(x;t)$ *works as an inequality index evaluated at fictitious income distribution*

$$y \equiv (\underbrace{\frac{x_1}{t_1},...,\frac{x_1}{t_1}}_{t_1},...,\underbrace{\frac{x_L}{t_L},...,\frac{x_L}{t_L}}_{t_L}).$$

Proof: See Appendix

The above proposition claims that the set of axioms 1-4 put forward previously for measuring the spatial concentration of economic activity plays a similar role to the set of basic properties commonly invoked for measuring income inequality (Shorrocks, 1984; Foster, 1985).

Next, we show that if axiom 5, *aggregation*, is added to the above list, geographic concentration indexes are completely characterized.

[12] The analysis is parallel to that followed by Alonso-Villar and Del Río (2007) to measure occupational segregation.
[13] The axiom of "replication invariance," which is also named "population principle," means that if the economy is replicated several times, the inequality index should not change.

Proposition 2. *Let* I_c *be a continuous concentration index that takes a zero value when the distribution of the sector of study among locations coincides with that of the distribution of reference* (i.e., when $\frac{x_l}{X}=\frac{t_l}{T}$). *Then,* I_c *is a concentration index satisfying axioms 1, 2, 3, 4, and 5 if and only if it can be written as an increasing monotonic transformation of index*

$$\Psi_\alpha(x;t)=\begin{cases}\frac{1}{\alpha(\alpha-1)}\sum_l \frac{t_l}{T}\left[\left(\frac{x_l/X}{t_l/T}\right)^\alpha-1\right] & \text{if } \alpha\neq 0,1\\ \sum_j \frac{x_l}{X}\ln\left(\frac{x_l/X}{t_l/T}\right) & \text{if } \alpha=1\end{cases},$$

where parameter α *represents concentration aversion.*[14]

Proof: See Appendix.

This family is a variant of the well-known generalized entropy family of inequality indexes. An advantage of these concentration indexes is that they can be additively decomposed, which is useful for undertaking detailed analyses of spatial patterns (Brülhart and Traeger, 2005; Alonso-Villar and Del Río, 2008b).

Note that even though the "relative" inequality measures have been the most popular in the field of income distribution because they are not affected by currency unit, the axiom of unit consistency recently proposed by Zheng (2007) has opened the door to the use of other inequality measures. In particular, the variance is an "absolute" inequality measure satisfying at the same time unit consistency and decomposability, which makes it an eligible option to measure the spatial concentration of economic activity. This index can be adapted to measure concentration as follows:

$$\Phi(x;t)=\frac{1}{T}\sum_l \frac{t_l}{T}\left[\frac{x_l}{t_l}-\frac{X}{T}\right]^2.$$

It is easy to see that concentration index Φ satisfies axioms 1, 3' and 4. In addition, it satisfies axiom 2, since function $f(z)=z^2$ is an increasing convex function, which implies

[14] If we had considered concentration indexes defined on the space of employment distributions $(x;t)$ where all components of vector x were strictly positive, rather than positive, then another index would have appeared: $\Psi_\alpha(x;t)=\sum_l \frac{t_l}{T}\ln\left(\frac{t_l/T}{x_l/X}\right)$ if $\alpha=0$.

that $\frac{t_i}{T}\left[\frac{x_i-d}{t_i}-\frac{X}{T}\right]^2+\frac{t_h}{T}\left[\frac{x_h+d}{t_h}-\frac{X}{T}\right]^2>\frac{t_i}{T}\left[\frac{x_i}{t_i}-\frac{X}{T}\right]^2+\frac{t_h}{T}\left[\frac{x_h}{t_h}-\frac{X}{T}\right]^2$ if i and h are two locations such that $x_i<x_h$ and $t_i=t_h$. Therefore, if $(x';t')$ is obtained from $(x;t)$ through a disequalizing movement of the type described in axiom 2, then $\Phi(x';t')>\Phi(x;t)$. Finally, it is straightforward to prove that this concentration measure also satisfies the *translation invariance* axiom (axiom 3’’), which is formally defined in our context in what follows.

Axiom 3’’: *Translation Invariance*. If employment in the sector of study increases (or decreases) in such a way that the change, a, is distributed across locations according to their employment weights in the distribution of reference, i.e., $(x';t')=\left(x_1+a\frac{t_1}{T},...,x_L+a\frac{t_L}{T};t\right)$, and $x_l+a\frac{t_l}{T}\le t_l\ \forall l$, then the concentration level of the sector should not change, i.e., $I_c(x';t)=I_c(x;t)$.

As a consequence of the above axiom, if employment in the chemicals industry increases, and this surplus is distributed among locations in such a way that if in a location overall manufacturing employment doubles that of another location, the former location receives twice as much of the extra employment in chemicals as the latter, then, the spatial concentration of the chemicals industry should not change. It follows, therefore, that the *translation invariance* axiom and the *scale invariance* axiom differ regarding the type of increments in the sector of study that are considered to be concentration-invariant. It is important to know which type of concentration invariance we prefer using and, therefore, which measure we should single out in order to measure the spatial concentration of production, since results can vary considerably.

Note that even though index $\Phi(x;t)$ has been obtained by extending an “absolute” inequality measure, it is actually a *relative* concentration measure, since it quantifies how much the distribution of the sector across locations, x, departs from the distribution of reference, t.

An advantage of the aforementioned index is that it is additively decomposable. Following the decomposition used in the literature on income distribution (Chakravarty, 2001), index $\Phi\left(x;t\right)$ can be decomposed as

$$\Phi(x;t) = \sum_{k} \frac{T_k}{T} \Phi(x^k;t^k) + \Phi(X_1,...,X_K;T_1,...,T_k) ,$$

when location units (regions) are grouped into K classes (countries). This decomposition of total concentration in the within (first addend) and between (second addend) components is analogous to the one corresponding to the generalized entropy family.

From all the above, it follows that apart from the concentration measures derived from the generalized entropy family, other inequality-based measures satisfying alternative invariance properties can be used to determine the spatial concentration of economic activity.

3. Employment Lorenz curves

In this section, we first show that *symmetry in locations, movement between locations, scale invariance,* and *insensitivity to proportional subdivisions of locations* (i.e., axioms 1-4) are the basic properties of the concentration measurement behind employment Lorenz curves, since any concentration measure satisfying these axioms is consistent with non-crossing employment Lorenz curves. Second, two decompositions of these curves are presented: one by groups of locations, and the other by subsectors. The first decomposition is similar to the one proposed by Bishop et al. (2003) to decompose the traditional Lorenz curve by population subgroups, while the second decomposition has no parallel in that paper but in Alonso-Villar and Del Río (2008a) in a context of occupational segregation.

3.1 Relationship between concentration measures and employment Lorenz curves

The Lorenz curve of the employment distribution of a sector is usually constructed as follows. First, locations are lined up in ascending order of the ratio of the Hoover-Balassa index

$\frac{x_l/X}{t_l/T}$, which is equivalent to ranking according to $\frac{x_l}{t_l}$.[15] Next, the cumulative proportion of aggregate employment, $\sum_{i\leq l}\frac{t_i}{T}$, is plotted on the horizontal axis and the cumulative proportion of employment in the sector of study, $\sum_{i\leq l}\frac{x_i}{X}$, is plotted on the vertical axis. Therefore, if we denote by $\tau_l \equiv \sum_{i\leq l}\frac{t_i}{T}$ the proportion of cumulative aggregate employment represented by the first *l* locations ranked according to the above criterion, the employment Lorenz curve can be written as follows:

$$L_{(x;t)}(\tau_l) = \frac{\sum_{i\leq l} x_i}{X}.$$

The first decile of the distribution represents 10% of aggregate employment, and it includes those locations where the sector of study has the lowest relative presence. The second cumulative decile represents 20% of aggregate employment, and it also includes locations where the sector has the lowest relative presence, and so on. Each point of the employment Lorenz curve indicates the proportion of employment in the sector corresponding to each cumulative decile of aggregate employment. In other words, the curve shows the under-representation of the sector with respect to aggregate employment, decile by decile. In the case where a sector of study was distributed across locations in the same manner as the distribution of reference, the employment Lorenz curve would be equal to the bisector and no concentration would exit.

As with standard Lorenz curves, we can say that distribution $(x;t)\in D$ *dominates in geographic concentration* distribution $(x';t')\in D$ if the employment Lorenz curve of the former lies at no point below the latter and at some point above.

In what follows, we formally establish the basic properties underlying the measurement of spatial concentration by using the employment Lorenz curves.[16]

[15] See, for example, Brülhart (2001). Alternatively, Krugman (1991) and Amiti (1999) ranked locations in descending order, but it is the ascending order that is consistent with income distribution literature.
[16] The analysis is similar to that put forward by Alonso-Villar and Del Río (2007) in a context of occupational segregation.

Proposition 3. Distribution $(x;t)$ *dominates in geographic concentration distribution* $(x';t')$ *if and only if* $I_c(x;t) < I_c(x';t')$ *for any concentration index* I_c *satisfying symmetry in locations, movement between locations, scale invariance, and insensitivity to proportional subdivisions of locations (i.e., axioms 1-4).*

Proof: See Appendix

The generalized entropy family of indexes defined in the previous section satisfies axioms 1-4 (Proposition 2) and therefore, by Proposition 3, it is consistent with non-crossing employment Lorenz curves. It is easy to see that an adequate version of the classic Gini index also satisfies axioms 1-4 and, therefore, it works as a concentration measure consistent with non-intersecting employment Lorenz curves:

$$G = \frac{\sum_{l,l'} \frac{t_l}{T}\frac{t_{l'}}{T}\left|\frac{x_l}{t_l} - \frac{x_{l'}}{t_{l'}}\right|}{2\frac{X}{T}}.$$

In income distribution analyses, the Gini index takes a zero value when all individuals have the same income, which represents egalitarian distribution, while it is equal to one if a single individual accumulates the total income of the economy. When using this index to measure geographic concentration, we should keep in mind that at the equalitarian distribution, i.e., if the sector of study is distributed across locations in the same way as the distribution of reference ($\frac{x_l}{X} = \frac{t_l}{T} \quad \forall l$), the Gini index is also equal to zero. However, if the employment of the sector is clustered at a single location, for example at location 1, the Gini index is not equal to one, as usually claimed, but to $\frac{T-t_1}{T}$. Therefore, the value of the index is higher, the lower the share of aggregate employment represented in the location of the sector.

As a consequence of Proposition 3, if the employment Lorenz curve corresponding to distribution $(x;t)$ dominates that of distribution $(x';t')$, the Gini index and any of the indexes of the generalized entropy family would have a higher value at the latter distribution, which makes the measurement of spatial concentration by using employment Lorenz curves a rather robust procedure. In other words, the properties behind the employment Lorenz curve are the basic agreement among these indexes. Therefore, the Lorenz criterion, when conclusive, has the advantage of allowing comparisons among spatial distributions by using the lesser number of value judgments, which makes this measurement rather appealing. However, even when curves do not cross, if one is interested in quantifying the differences between the concentration levels of two distributions, the use of these indexes can be useful, even though we should be aware of the fact that they differ in terms of additional properties, since each of them focuses on a different aspect of the distribution.

3.2 Decomposing employment Lorenz curves

While additive decompositions of the generalized entropy indexes have been proposed in the literature of industrial concentration, as far as we know, no decompositions of the employment Lorenz curves have yet been suggested. In what follows, we offer two forms of decomposition of these curves: one by groups of locations, and the other by subsectors.

Proposition 4. Assume that locations can be classified into *K* mutually exclusive groups so that the distributions x and t can be expressed as $(x;t)=(x^1,...,x^K;t^1,...,t^K)$, where x^k denotes the employment distribution of the sector across locations in group *k*, and t^k is that of aggregate employment ($k=1,...,K$). Then, the employment Lorenz curve, $L_{(x;t)}$, can be decomposed as follows:

$$L_{(x;t)}(\tau_l)=\sum_k \frac{X_k}{X}\tilde{L}_{(\tilde{x}^k;t)}(\tau_l),$$

where X_k is the employment level of the sector in group k, $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$ is like the employment Lorenz curve of distribution $(\tilde{x}^k;t)$ except that locations are ranked according to ratios $\frac{x_l}{t_l}$, and $\tilde{x}^k$ is an L-dimensional vector resulting from enlarging vector x^k with zero-values for those locations that are not included in group *k*.

Proof:

Define indicator G_l^k so that $G_l^k=1$ if location (region) *l* belongs to group (country) *k* and $G_l^k=0$ otherwise ($l=1,...,L$, and $k=1,...,K$). By using vector x^k, we can build $\tilde{x}^k$ as follows $\tilde{x}^k=(x_1G_1^k,...,x_LG_L^k)$. In other words, $\tilde{x}^k$ is a fictitious employment distribution having the same dimension as the original distribution x so that it can be compared to the distribution of total employment *t*. By keeping locations ranked in ascending order of the ratios $\frac{x_l}{t_l}$ ($l = 1,...,L$), and maintaining $\tau_l \equiv \sum_{i \le l} \frac{t_i}{T}$, we could define $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$ as the proportion of employment in the sector corresponding to the locations ranked before l that are included in group k. In other words, $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l) = \frac{\sum_{i \le l} x_i G_i^k}{X_k}$. Then, the employment Lorenz curve corresponding to distribution $(x;t)$ can be decomposed as $L_{(x;t)}(\tau_l) = \frac{\sum_{i \le l} x_i}{X} = \sum_k \frac{X_k}{X} \frac{\sum_{i \le l} x_i G_i^k}{X_k} = \sum_k \frac{X_k}{X} \tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$, which completes the proof. □

Consequently, the expression $\frac{X_k}{X} \frac{\tilde{L}_{(\tilde{x}^k;t)}(\tau_l)}{L_{(x;t)}(\tau_l)}$ measures the contribution of group k to the value of the employment Lorenz curve in the corresponding percentile. Assume, for example, that our location units are the European regions and that we are interested in grouping them by country. Consider again that we focus on the chemicals sector. As mentioned above, the first decile of the employment distribution includes those regions where the chemicals industry has the lowest relative presence, and it accounts for 10% of manufacturing employment in Europe. By using the above decomposition for the first decile, one could determine whether the regions with the lowest employment in chemicals belong to Spain, France, Italy, etc.

On the other hand, function $\tilde{L}_{(\tilde{x}^k;t)}$ also enables one to determine how the sector of study in country k is distributed among deciles by using expression $\tilde{L}_{(\tilde{x}^k;t)}(\tau_l + 0.1) - \tilde{L}_{(\tilde{x}^k;t)}(\tau_l)$, which indicates the proportion of employment in the sector in country k in each (non-cumulative) decile. This analysis would permit one, for example, to find out whether the distribution of chemicals in France across the deciles of manufacturing employment in Europe differs from that of Germany.

Next, without loss of generality, let employment in the sector be classified into two mutually exclusive subsectors, A and B, so that $(x_1,...,x_L) = (x_1^A + x_1^B,...,x_L^A + x_L^B)$. Denote by X^A (respectively X^B) the employment level of subsector A (respectively B).

Proposition 5. If the sector can be partitioned into two mutually-exclusive subsectors *A* and *B* so that $(x;t)=(x^A+x^B;t)$, then the employment Lorenz curve, $L_{(x;t)}$, can be decomposed as follows:

$$L_{(x;t)}(\tau_l)=\frac{X^A}{X}\tilde{L}_{(x^A;t)}(\tau_l)+\frac{X^B}{X}\tilde{L}_{(x^B;t)}(\tau_l),$$

where $\tilde{L}_{(x^A;t)}(\tau_l)$ is like the employment Lorenz curve corresponding to $(x^A;t)$, and $\tilde{L}_{(x^B;t)}(\tau_l)$ is like the employment Lorenz curve corresponding to $(x^B;t)$, except that locations have been ranked according to ratios $\frac{x_l}{t_l}$.

Proof:

Let us define $\tilde{L}_{(x^A;t)}(\tau_l)=\frac{\sum_{i\le l} x_i^A}{X^A}$ and $\tilde{L}_{(x^B;t)}(\tau_l)=\frac{\sum_{i\le l} x_i^B}{X^B}$. The proof is immediate by noting that the employment Lorenz curve corresponding to distribution $(x;t)$ can be decomposed as

$$L_{(x;t)}(\tau_l)=\frac{X^A}{X}\frac{\sum_{i\le l} x_i^A}{X^A}+\frac{X^B}{X}\frac{\sum_{i\le l} x_i^B}{X^B}=\frac{X^A}{X}\tilde{L}_{(x^A;t)}(\tau_l)+\frac{X^B}{X}\tilde{L}_{(x^B;t)}(\tau_l).$$ □

This decomposition can also be easily generalized to more than two subsectors so that expression $\frac{X^A}{X}\frac{\tilde{L}_{(x^A;t)}(\tau_l)}{L_{(x;t)}(\tau_l)}$ measures the contribution of subsector *A* to the employment Lorenz curve of the sector in each cumulative decile. This analysis would permit one, for example, to determine whether in the first decile chemicals employment corresponds mainly to pharmaceutical products, manufacture of pesticides and other agrochemical products, manufacture of manmade and synthetic fibers, etc.

Furthermore, expression $\tilde{L}_{(x^A;t)}(\tau_l+0.1)-\tilde{L}_{(x^A;t)}(\tau_l)$ enables one to determine how subsector *A* is distributed among (noncumulative) deciles. In particular, this would allow one to determine whether a given subsector of the chemicals industry is located mainly in regions where the chemicals industry has a high presence or, on the contrary, it follows a different location pattern than the sector as a whole. It also allows, for example, studying whether the distribution of pharmaceutical products among European regions differs from that of manufactures of manmade and synthetic fibers.

4. Conclusions

This paper has formally shown the parallel that exists between inequality measurement and spatial concentration measurement. As far as we know, this is the first time that this connection is formally established, since up to now inequality indexes have been adapted to spatial analysis without an axiomatic discussion. This examination has allowed us to unveil the properties that the literature on regional economics is implicitly assuming when using inequality measures to quantify spatial concentration of economic activity. In particular, this paper has shown the properties satisfied by the Gini index and the generalized entropy family when using them to analyze location patterns. In addition, another inequality-based concentration measure has been proposed for measuring geographic concentration of economic activity. It has been shown that this concentration index satisfies an alternative concentration-invariance condition and also that it is additively decomposable, which is very helpful for empirical analyses.

Even though additive decompositions of the generalized entropy family of indexes have been proposed in the literature to measure industrial concentration, as far as we know, no decompositions of the employment Lorenz curves have yet been suggested. For this reason, this paper has finally offered two decompositions of these curves. One is obtained when locations are partitioned into different groups, while the other is obtained by classifying the sector into several subsectors.

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Appendix

Proof of Proposition 1

In what follows we prove that if the concentration index I_c satisfies axioms 1-4, then index I evaluated at the fictitious income distribution as $I(y) := I_c\left(x;t\right)$, where $y \equiv \left(\underbrace{\frac{x_1}{t_1},...,\frac{x_1}{t_1}}_{t_1},...,\underbrace{\frac{x_L}{t_L},...,\frac{x_L}{t_L}}_{t_L} \right)$, works as an inequality index satisfying scale invariance, symmetry, the Pigou-Dalton principle, and replication invariance .

a) I is well defined. Note that several vectors $\left(x;t\right)$ can be reached after grouping individuals in the fictitious "income distribution" who belong to the same location depending on how many locations are considered. However, by axiom 4, all these vectors have the same spatial concentration level, since they can be obtained from each other by proportional subdivisions.

b) Scale invariance. In the literature of income distribution, an index is said to satisfy this property if and only if inequality remains constant when multiplying all incomes by the same positive scalar. This property is certainly satisfied by index I since $I(\theta\frac{x_1}{t_1},...,\theta\frac{x_1}{t_1},...,\theta\frac{x_L}{t_L},...,\theta\frac{x_L}{t_L}) = I_c(\theta x;t)$, which is equal to $I_c(x;t)$ because I_c satisfies axiom 3 (case where $a > 0,\ b = 1$).

c) Symmetry. This property requires that individuals play symmetric roles in the inequality index. This is satisfied by I since I_c satisfies axioms 1 and 4.

d) The Pigou-Dalton transfer principle. Any regressive transfer in this fictitious economy corresponds to a situation where a location *i* transfers employment in the sector of study to another location *k*, where $t_i = t_h$ and $x_i < x_h$. Since I_c satisfies axiom 2, the second situation leads to a higher concentration index and, therefore, to a higher value of I_c . As a consequence, the regressive transfer leads to higher inequality.

e) Replication invariance. This means that when replicating the economy k-times, so that for every individual in the previous economy there are now *k* identical individuals, income inequality is not altered. This axiom is satisfied here since a k-replication of the fictitious distribution leads to a k-replication of vector $(x;t)$, and I_c satisfies axiom 3 (case where $a=b$). □

Proof of Proposition 2

First step: *Any concentration index* I_c *satisfying axioms 1-5 can be written as a strictly increasing monotonic transformation of* Ψ_α.

Following Shorrocks (1984) and Foster (1985), any continuous inequality measure I taking a zero value at the egalitarian distribution and satisfying scale invariance, replication invariance, the Pigou-Dalton transfer principle, symmetry, and aggregation can be written as $I(y)=F^{-1}(I_\alpha(y))$ for some parameter α, where F is a strictly increasing function such that $F:[0,\infty)\to\mathbb{R}$, with $F(0)=0$ and I_α is the well-known generalized entropy family of inequality indexes:

$$I_\alpha(y)=\begin{cases}\dfrac{1}{n\alpha(\alpha-1)}\sum_i\left[\left(\dfrac{y_i}{\frac{1}{n}\sum_k y_k}\right)^\alpha-1\right] & \text{if } \alpha\neq 0,1\\[2ex] \dfrac{1}{\mathrm{n}}\sum_i\left[\dfrac{y_i}{\frac{1}{n}\sum_k y_k}\ln\left(\dfrac{y_i}{\frac{1}{n}\sum_k y_k}\right)\right] & \text{if } \alpha=1\\[2ex] \dfrac{1}{\mathrm{n}}\sum_i\ln\left(\dfrac{\frac{1}{n}\sum_k y_k}{y_i}\right) & \text{if } \alpha=0\end{cases}$$

In Proposition 1 we proved that any concentration index I_c satisfying axioms 1-4 can be regarded as an inequality index I satisfying scale invariance, symmetry, the Pigou-Dalton transfer principle and replication invariance. It is easy to see that if I_c is a continuous function, so too is I. If we additionally show that I is aggregative and also that it is equal to zero at the equalitarian distribution, we can use Shorrocks's result in order to characterize inequality index I.

An inequality index I is defined as aggregative if $I(y)=A(I(y^1),\mu(y^1),n(y^1),I(y^2),\mu(y^2),n(y^2))$, where A is a continuous function that is strictly increasing in the first and fourth arguments, y^i represents the income distribution corresponding to individuals' group i, $\mu(.)$ is the average of the corresponding distribution, and $n(.)$ is the number of individuals in the corresponding group. In our case, the "income" distribution is $y\equiv(\underbrace{\frac{x_1}{t_1},...,\frac{x_1}{t_1}}_{t_1},...,\underbrace{\frac{x_L}{t_L},...,\frac{x_L}{t_L}}_{t_L})$, and the average of that distribution is equal to $\frac{X}{T}$.

In what follows, we show that our I is an aggregative inequality index. For the sake of simplicity, assume that class 1 includes locations $l=1,...,i$, while class 2 is the complementary. By definition

$$I\left(\underbrace{\frac{x_1}{t_1},...,\frac{x_1}{t_1},...,\frac{x_i}{t_i},...,\frac{x_i}{t_i}}_{\text{class 1}}\underbrace{\frac{x_{i+1}}{t_{i+1}},...,\frac{x_{i+1}}{t_{i+1}},...,\frac{x_L}{t_L},...,\frac{x_L}{t_L}}_{\text{class 2}}\right)=I_c(x;t)\cdot$$

On the other hand, since by axiom 5 I_c is an aggregative concentration index:

$$I_c(x;t)=I_c(x^1,x^2;t^1,t^2)=A\left(I_c(x^1;t^1),\frac{X_1}{T_1},T_1,I_c(x^2;t^2),\frac{X_2}{T_2},T_2\right).$$

Note that $I_c(x^1;t^1)=I(\frac{x_1}{t_1},...,\frac{x_1}{t_1},...,\frac{x_i}{t_i},...,\frac{x_i}{t_i})$, and $I_c(x^2;t^2)=I(\frac{x_{i+1}}{t_{i+1}},...,\frac{x_{i+1}}{t_{i+1}},...,\frac{x_L}{t_L},...,\frac{x_L}{t_L})$. Therefore,

$$I\left(\underbrace{\frac{x_1}{t_1},...,\frac{x_1}{t_1},...,\frac{x_i}{t_i},...,\frac{x_i}{t_i}}_{\text{class 1}}\underbrace{\frac{x_{i+1}}{t_{i+1}},...,\frac{x_{i+1}}{t_{i+1}},...,\frac{x_L}{t_L},...,\frac{x_L}{t_L}}_{\text{class 2}}\right)=A\left(I(\frac{x_1}{t_1},...,\frac{x_1}{t_1},...,\frac{x_i}{t_i},...,\frac{x_i}{t_i}),\frac{X_1}{T_1},T_1,I(\frac{x_{i+1}}{t_{i+1}},...,\frac{x_{i+1}}{t_{i+1}},...,\frac{x_L}{t_L},...,\frac{x_L}{t_L}),\frac{X_2}{T_2},T_2\right),$$

where $\frac{X_1}{T_1}$ (respectively, $\frac{X_2}{T_2}$) represents the average "income" of "individuals" in class 1 (respectively, 2), while T_1 (respectively, T_2) is the number of "individuals" in that class. Therefore, the inequality index I is aggregative.

Finally, note that I is equal to zero when all "individuals" have the same "income," i.e., when all locations have the same employment shares in the sector under consideration (i.e., when $\frac{x_l}{t_l}=\frac{X}{T}$ $\forall l$).

Therefore, by using Shorrocks's result, it follows that $I(y)=F^{-1}(I_\alpha(y))$ for $\alpha \neq 0,1$ or $\alpha=1$.[17] On the other hand, $I_c(x;t)=I(y)$ and $F^{-1}(I_\alpha(y))=F^{-1}(\Psi_\alpha(x;t))$, which completes the proof of step one.

Second step: $F^{-1}(\Psi_\alpha)$ *is a concentration index satisfying symmetry in locations, movement between locations, scale invariance, insensitivity to proportional subdivisions of locations, and aggregation.*

In order to prove this, it suffices to show that Ψ_α satisfies the above properties, which is done in what follows. It is immediate proven that Ψ_α verifies *scale invariance*, *symmetry in locations*, and *insensitivity to proportional subdivisions*. To demonstrate that Ψ_α satisfies the axiom of *movement between locations*, note that any disequalizing movement from location *i* to *h*, where $t_i=t_h$ and $x_i<x_h$, implies moving from "income" distribution $y=\left(\frac{x_1}{t_1},...,\frac{x_1}{t_1},...,\frac{x_i}{t_i},...,\frac{x_i}{t_i},...\frac{x_h}{t_h},...,\frac{x_h}{t_h},...,\frac{x_L}{t_L},...,\frac{x_L}{t_L}\right)$ to "income" distribution $y'=\left(\frac{x_1}{t_1},...,\frac{x_1}{t_1},...,\frac{x_i-d}{t_i},...,\frac{x_i-d}{t_i},...,\frac{x_h+d}{t_h},...,\frac{x_h+d}{t_h},...,\frac{x_L}{t_L},...,\frac{x_L}{t_L}\right)$. On the other hand, $I_\alpha(y)=\Psi_\alpha(x;t)$ and $I_\alpha(y')=\Psi_\alpha(x';t')$. Since I_α is an inequality measure satisfying the Pigou-Dalton transfer principle and *y'* can be obtained from *y* by a finite sequence of regressive transfers it follows that $\Psi_\alpha(x';t')>\Psi_\alpha(x;t)$. Next, we prove that Ψ_α is aggregative. By simple calculations Ψ_α can be written as

$$\Psi_\alpha(x^1,x^2;t^1,t^2)=\begin{cases}-\frac{1}{\alpha(\alpha-1)}+\left[\begin{array}{l}\left(\frac{T_1}{T}\right)^{1-\alpha}\left(\frac{X_1}{X}\right)^{\alpha}\left(\Psi_\alpha\left(x^1;t^1\right)+\frac{1}{\alpha(\alpha-1)}\right)+\\+\left(\frac{T_2}{T}\right)^{1-\alpha}\left(\frac{X_2}{X}\right)^{\alpha}\left(\Psi_\alpha\left(x^2;t^2\right)+\frac{1}{\alpha(\alpha-1)}\right)\end{array}\right] & \text{for } \alpha\neq 0,1\\ \frac{X^1}{X}\left[\Psi_\alpha(x^1;t^1)+\ln\left(\frac{X_1}{T_1}\frac{T}{X}\right)\right]+\frac{X_2}{X}\left[\Psi_\alpha(x^2;t^2)+\ln\left(\frac{X_2}{T_2}\frac{T}{X}\right)\right] & \text{for } \alpha=1\end{cases}$$

[17] The case where $\alpha=0$ is discarded, because when the sector of study has no employment in location *l* (i.e., when $x_l=0$) and $\alpha=0$, the index value would be infinite and, therefore, have no sense. The case where $\alpha=1$ does not have the same problem since $\lim_{x_j\to 0}\frac{x_j/X}{t_j/T}\ln\left(\frac{x_j/X}{t_j/T}\right)=0$.

On the other hand, $T = T_1 + T_2$ and $X = X_1 + X_2$. Therefore, Ψ_α can be written as $\Psi_\alpha(x^1, x^2; t^1, t^2) = A\left(\Psi_\alpha\left(x^1; t^1\right), \frac{X_1}{T_1}, T_1, \Psi_\alpha\left(x^2; t^2\right), \frac{X_2}{T_2}, T_2\right)$, which completes the proof. □

Proof of Proposition 3

From Proposition 1, any concentration index I_c satisfying axioms 1-4 leads to an inequality index satisfying scale invariance, symmetry, the Pigou-Dalton transfer principle and replication invariance. On the other hand, given the relationship between Lorenz curves and relative inequality measures established by Foster (1985), the Lorenz curve of a distribution dominates another if and only if any inequality measure satisfying the above four basic properties takes a lower value at the former distribution. Since the Lorenz curve for employment distribution $\left(x;t\right)$ is like the Lorenz curve for our "income" distribution $y \equiv (\frac{x_1}{t_1}, ..., \frac{x_1}{t_1}, ..., \frac{x_L}{t_L}, ..., \frac{x_L}{t_L})$, from all the above it follows that $I_c\left(x;t\right) < I_c\left(x';t'\right)$ if and only if the former distribution Lorenz dominates the latter, which completes the proof. □

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